

1811000101020001
B. C. A. F.Y. (Sem -I)
Examination December-2023
Mathematics - I

Seat No:

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[Time: Three Hours]

[Max. marks: 70]

Instructions

- (1) All questions are compulsory
 (2) Figure to the right indicates full marks of the questions

Student's Signature

Q.1 Answer the following

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- 1) Define: Equal set
- 2) Define: Function
- 3) If $A = \{10, 12, 14, 16, 8, 2\}$ And $B = \{18, 12, 14, 11, 10, 6\}$ then find $B - A$.
- 4) Define: Conjunction
- 5) Write the law of tautology and contradiction.
- 6) Evaluate: $\begin{vmatrix} x+y & x \\ x & x-y \end{vmatrix}$
- 7) Give two example of dual statement.
- 8) If $f(x) = \frac{x^2+1}{x^3-x+1}$ then find $f(1) - f(0)$.
- 9) Define: Transpose matrix
- 10) If $A = \{3, 4\}$ then find A^2 .

Q.2 A) In usual notations prove that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

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OR

A) State and prove DE MORGANS law for union.

B) Attempt any two.

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- 1) If $A = \{x: x \leq 3; x \in N\}$, $B = \{x: 1 < x \leq 5; x \in N\}$, & $C = \{x: x \text{ is an even positive integer less than } 10\}$ then prove that
 $A \cap (B - C) = (A \cap B) - (A \cap C)$.
- 2) Prove that: $A - (B \cup C) = (A - B) \cap (A - C)$
- 3) In a class of 42 students, each play at least one of the three games Cricket, Hockey and Football. It is found that 14 play Cricket, 20 play Hockey and 24 play Football, 3 play both Cricket and Football, 2 play both Hockey and Football. None play all the three games. Find the number of students who play Cricket but not Hockey.

- 4) If $A = \{2,3,4\}, B = \{1,3,4\}, S = \{1,2,3\}, T = \{1,3,5\}$ verify that
 $(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$.

Q.3 A) If $A = \{1,2\}, B = \{2,3\}, C = \{3,5\}$ then prove that

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

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OR

A) If $f(x) = \frac{x(x-2)}{x-1}$ then find $f(0) + f(-1) + f(3) - f(2)$

B) Attempt any two.

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- 1) The fixed cost of a factory producing particular types of bag is Rs. 9000 and the variable cost per bag is Rs. 110. If the selling price per bag is Rs. 240 then find the profit function.
- 2) If $f: Z - \{-1\} \rightarrow Z$ where $f(x) = \frac{x^3+1}{x+1}$ and $g: Z \rightarrow Z$ where $g(x) = x^2 - x + 1$ then examine whether the functions are equal.
- 3) The total cost and total Revenue functions are given as $C(x) = 5x + 350$ and $R(x) = 50x - x^2$ then find the profit for $x = 10$.
- 4) The cost of manufacturing 1000 pens in Rs. 17,000 and that of manufacturing 1,500 pens is 22,000. If the relationship between the number of units produced and its cost is linear, find the relation and also find the cost of manufacturing 2000 pens.

Q.4

A) Prove that $\begin{vmatrix} (a-1)^2 & (b-1)^2 & (c-1)^2 \\ 1 & 1 & 1 \\ a+1 & b+1 & c+1 \end{vmatrix} = (a-b)(b-c)(c-a)$

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OR

A) If $A = \begin{bmatrix} 7 & 3 & -5 \\ 0 & 4 & 2 \\ 1 & 5 & 4 \end{bmatrix}$ find the matrix $D = 2A + B - C$ and $B = 3A$.

B) Attempt any two.

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- 1) If $A = \begin{bmatrix} 2 & 6 \\ 7 & 2 \end{bmatrix}, B = \begin{bmatrix} -3 & 5 \\ 0 & 8 \end{bmatrix}, C = \begin{bmatrix} 4 & 7 \\ 9 & 5 \end{bmatrix}$ then prove that $A(BC) = (AB)C$.
- 2) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$
- 3) Solve the following equations by Cramer's Rule:
 $2x + 8y = 3xy$
 $4x + 12y = 5xy$

4) Find AB and BA for the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$ AND $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

Q.5 A) Construct the input/output table for $f: B^3 \rightarrow B, f(x_1, x_2, x_3) = (x_1 \cdot x'_2) + x_3$. **5**

OR

A) Using De-Morgan's laws give the negations of the following:

- He is a doctor and he is a mathematician.
- $x \notin A$ or $x \in B$

B) **Attempt any two.**

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- Simplify the Boolean expression: $x + x' \cdot (x + y) + y \cdot z$
- Construct the input/output table for $f(x_1, x_2, x_3) = (x_1 \cdot x'_2) \cdot x_3$
- Show that D_8 is a Boolean algebra. Where for all element of D_8 ;

$$x + y = \text{lcm of } x \text{ \& } y; x \cdot y = \text{gcd of } x \text{ \& } y; x' = \frac{21}{x}$$

4) Prepare truth table for the following:

- $(p \wedge q) \Rightarrow r$
- $[(\sim p) \vee q] \Rightarrow \sim q$