1811000101020001
F.Y. B.C.A. (Sem – I)
Examination December-2022
Mathematics

Seat No:

Student's Signature

[Time: Three Hours] **[Max. Marks:70]**

Instructions:
(1) All questions are compulsory.
(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

Q.1 Answer the following: **[10]**

1] Define Infinite Set with illustration.
2] If A = {a, e, i, o, u} then find the number of sets in power set of A.
3] Define: One-One function.
4] If $f : A \rightarrow B$, where $f(x) = 2x - 1$ and A = {1, 2, 3, 4} then find R_f.
5] Define: Critical row.
6] Prove that $p \wedge c = c$ where p is the statement and c means contradiction.
7] Define: Principle of duality.
8] Write the dual statement of $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$
9] Define Minor of an element.
10] Mention which type of matrix is $A = [1 \quad 2 \quad 3 \quad 4]$

Q.2 (a) In usual notations prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. **[05]**

OR

(a) In usual notations prove that $(A \cap B)' = A' \cup B'$

(b) Attempt **any two:** **[10]**

1] In a society of 250 people, each person speaks either Gujarati or Hindi, 158 people speak Gujarati and 185 speak Hindi then find
(i) the number of people who can speak both the languages.
(ii) the number of people who speak only Gujarati.

2] If A = {1, 3, 4, 6}, B = {2, 4, 5} and C = {3, 4, 5} then verify that $A \cap (B - C) = (A \cap B) - (A \cap C)$

3] If $A = \{x : x \in N, x < 6\}$, $B = \{x : x \in N, 3 < x < 9\}$ and C = {1, 4, 5, 7} the verify $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

4] If A = {2, 3, 5, 6, 8), B = {1, 3, 5, 7, 9} and U = {1, 2,..., 10} find $A \cup B$, $A' \cup B'$, $A - B$, $B \times A$.

Q.3 **(a)** If $f(x) = x^2 + x - 1$, find the value of $f(x+1) - 3f(x-1) + 2f(x)$ **[05]**

OR

(a) If $f(x) = \frac{1}{x} + \frac{2}{x-3}$ then find $f(1), f(2), f\left(\frac{1}{3}\right), f(-3)$.

(b) Attempt **any two:** **[10]**

1] If $f(x) = \frac{x^3-27}{x-3}$ then find $\frac{f(-1)+f(-2)}{f(2)+f(0)}$

2] If $f(x) = 2x^2 - 1$ and $g(x) = 2x - 1$; where $D_f = D_g = \{0, 1, 2\}$. Is $f = g$? Justify your answer.

3] If $f(x) = \frac{1}{x}, x \in Z - \{-1, 0, 1\}$ then prove that $f(x-1) - f(x+1) = \frac{2}{x^2-1}$

4] It is observed that a quadratic function $ax^2 + bx + c$ fits the data points (-1, 8), (1, 4) and (2, 5). Find the constants a, b and c and find y when x = 4.

Q.4 **(a)** $A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$ then check weather $A^2 = I$? **[05]**

OR

(a) If $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & -1 & 3 \\ -1 & -1 & 2 \end{bmatrix}$ then find $A^2 + 2A - I$

(b) Attempt **any two:** **[10]**

1] Solve the following equations by Cramer's Rule:

$$x + 2y + 3z - 14 = 0$$
$$2x + y + z - 7 = 0$$
$$5x + 2y + z - 12 = 0$$

2] If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 1 \\ 2 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 4 \\ 1 & -3 & 2 \\ -1 & 3 & 2 \end{bmatrix}$ then show that $(AB)^T = B^T A^T$.

3] Find inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & -8 \\ 6 & -3 & 0 \end{bmatrix}$

4] Solve the following equations by Cramer's Rule:

$$2x + 8y = 3xy$$
$$4x + 12y = 5xy$$

Q.5 **(a)** Show that $\{p \Rightarrow (q \Rightarrow r)\} \Rightarrow \{(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$ is a tautology. **[05]**

OR

(a) Show that $(D_{21}, +, \cdot, ', 1, 21)$ is a Boolean Algebra $\forall a, b \in D_{21}$

$a + b = \text{lcm of } a, b$

$a \cdot b = \text{gcd of } a, b$

$a' = 21/a$

(b) Attempt **any two:** **[10]**

1] Express Boolean function $f(a, b, c) = (a \cdot b) + (a \cdot c) + (b \cdot c)$ as a sum of product in three variables.

2] Using truth tables prove that $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$.

3] For the element x, y of a boolean algebra, prove that $x \cdot y' = 0 \Leftrightarrow x \cdot y = x$

4] Check the validity of the following argument:
Hypothesis $S_1 : p \Rightarrow q$, $S_2 : q \Rightarrow r$, $S_3 : p$
Conclusion: S: r