1811000101020001
BCA (Sem-I)
Examination June - 2023
Mathematics

Seat No:

Student's Signature

[Time: Three hours] **[Max. marks:70]**

Instructions: (1) All questions are compulsory.
(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

Q.1 Answer the following: **[10]**

1] Define: Proper subset and improper subset with illustrations.
2] If $f(x) = x^2 - x + 1$ then find f(2) − f(−2)
3] Define: Conjunction and Tautology.
4] In a Boolean Algebra prove that 0' = 1 and 1' = 0.
5] If $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ then find adj. A and obtain $|A|$

Q.2 (a) In usual notations prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **[05]**

OR

(a) In usual notations prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(b) Attempt **any two:** **[10]**

1] IF $A = \{x \mid x \in N;\ 2 < x < 6\}$, $B = \{x | x \in N;\ x^2 < 5x\}$ and $U = \{x | x \in N; x < 10\}$ then prove that $(A \cup B)' = A' \cap B'$
2] If $A = \{x: x \in N, x < 6), B = \{x: x \in N, 3 < x < 9\}$ and C = {1, 4, 5, 7} the verify $(AUB) \cap C = (A \cap C)\ U\ (B \cap C)$
3] In a college there are 500 students and out of them 300 have taken Economics and 250 have taken Statistics. All students have taken at least one of these two subjects. How many of them have taken both the subjects?

Q.3 (a) If $f(x) = x^2 + x - 1$, find the value of $f(x + 1) - 3f(x - 1) + 2f(x)$ **[05]**

OR

(a) If $f(x) = \frac{1}{x} + \frac{2}{x-3}$ then find f(1), f(2), f $(\frac{1}{3})$, f(−3).

(b) Attempt **any two:** **[10]**

1] It is observed that a quadratic function $ax^2 + bx + c$ fits the data points (-1, 8), (1, 4) and (2,5). Find the constants a, b and c and find y when x = 4.

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2] If $f(x) = \frac{x^3-27}{x-3}$ then find $\frac{f(-1)+f(-2)}{f(2)+f(0)}$

3] If $f(x) = 2x^2 - 1$ and $g(x) = 2x - 1$; where $D_f = D_g = \{0,1,2\}$. Is $f = g$? Justify your answer.

Q.4

(a) $A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$ then find that $A^2 + 2A - I$. **[05]**

OR

(a) If $A = \begin{bmatrix} 6 & 3 \\ -3 & 9 \\ 12 & -6 \end{bmatrix}$ find matrix B such that $2A^T + 3B = 0$

(b) Attempt **any two:** **[10]**

1] If $A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 6 & 8 \\ 5 & 0 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 & 9 \\ -3 & 0 & 2 \\ 7 & 6 & 5 \end{bmatrix}$ then show that $(A + B)^T = A^T + B^T$.

2] Solve the following equations by Cramer's Rule:
$\frac{x}{3} + \frac{y}{4} = 1, \frac{2x}{9} - \frac{y}{2} = 6$

3] Find inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & -8 \\ 6 & -3 & 0 \end{bmatrix}$

Q.5

(a) Show that $D_{21}, +, ., ', 1, 21)$ is a Boolean Algebra $\forall a, b \in D_{21}$ **[05]**
$a + b = 1$cmof a, b
$a.b =$ gcd of a, b
$a' = 21/a$

OR

(a) Check the validity of the following argument:
Hypothesis $S_1: p \Rightarrow q, S_2: \sim p$
Conclusion: S: q

(b) Attempt **any two:** **[10]**

1] Construct the input/output table for
$f: B^3 \rightarrow B, f(x_1, X_2, X_3) = (X_1 X_2') + X_3$

2] Using truth tables prove that $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$.

3] Express Boolean function $f(a, b, c) = (ab) + (ac) + (b.c)$ as a sum of product in three variables.

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