1811000101020001
First Year B.C.A (Sem-I)
Examination March -2023
Mathematics

Seat No:

Student's Signature

[Time: Three Hours] **[Max. Marks:70]**

Instructions:
1) All questions are compulsory.
2) Figures to the right indicate marks of corresponding question.
3) Follow usual notations.
4) Use of non-programmable scientific calculator is allowed.

Q.1 Answer the following: **10**

1) What are Equivalent sets?
2) If $A = \{1,2,3\}, B = \{1,2,3,4,5,6\}$ then how will set A be related to B?
3) If $f(x) = x^2 - x + 1$ then find f(0) + f(-1)
4) Define: Many-one function.
5) Define: Conjunction.
6) Define: Tautology.
7) In a Boolean Algebra prove that 0'=1 and 1'=0.
8) Define: Principle of duality.
9) Evaluate: $\begin{vmatrix} -6 & 2 \\ -3 & -4 \end{vmatrix}$
10) If $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ then find adj. A.

Q.2 a) In usual notations prove that $A \cap B = B \cap A$ **05**

OR

a) In usual notations prove that $(A \cap B) \cap C = A \cap (B \cap C)$. **10**

b) Attempt **any two**:

1) If $U = \{1,2,3,a,b,c\}, A = \{a,b,c\}, B = \{1,2,3\}, C = \{1,2,a\}, D = \{3,a,b\}$ then find
   i) $(A - B) \cap (B - A)$ and ii) $(C \cup D)'$

2) If $A = \{x/x \leq 3; x \in N\}, B = \{x/x < x \leq 5; x \leq N\}$ and C={x/x is an even positive integer < 10} then verify that $A \cap (B - C) = (A \cap B) - (A \cap C)$

3) If $A = \{1,3\}, B = \{3,5\}$ and $C = \{3,5,6\}$ the verify $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

4) If $A = \{x | x \in N; 2 < x < 6\}$, $B = \{x | x \in N; x^2 < 5x\}$ and $U = \{x | x \in N; x < 10\}$ then prove that $(A \cup B)' = A' \cap B'$

Q.3 a) If $f(x) = \frac{1}{x} + \frac{2}{x-3}; x \in R - \{0,3\}$ then find $f(1), f(2), f(-3), f\left(\frac{1}{2}\right)$ **05**

OR

a) If $f(x) = x(x+1)(2x+1)$ then prove that $f(x) - f(x-1) = 6x^2$.

b) Attempt **any two**: **10**

1) If $f(x) = x^3$ and $g(x) = 3x^2 - 2x$ where $D_f = D_g = \{0,1,2\}$. If f=g? Justify your answer.
2) It is observed that a quadratic function $ax^2 + bx + c$ fits the data points (1,9), (2,14) and (3,23). Find the constants a, b and c and find y when x=4.
3) If $f(x) = \frac{x^2 - x}{x+3}$ then find $\frac{f(0)+f(-2)}{f(1)+f(3)}$
4) If $f(x) = x^2 + 4x + 5$ and $g(x) = 2x + 1$ then prove that $f(1) - 2\,g(2) = 0$

Q.4 a) $A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$ then prove that $A^2 = I$. **05**

OR

a) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then prove that $A^2 - 4A - 5I = 0$

b) Attempt **any two**: **10**

1) If $A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 6 & 8 \\ 5 & 8 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 & 9 \\ -3 & 0 & 2 \\ 7 & 6 & 5 \end{bmatrix}$ them show that $(A+B)^T = A^T + B^T$.

2) Solve the following equations by Cramer's Rule:
$x + 2y + 3z - 14 = 0$
$2x + y + z - 7 = 0$
$5x + 2y + z - 12 = 0$

3) Solve the following equations by Cramer's Rule:
$x + 6y = 2xy$
$3x + 2y = 2xy$

4) Find inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & -8 \\ 6 & -3 & 0 \end{bmatrix}$

Q.5 a) Let $B = \{0,1\}$. Prepare an input/output table for the Boolean function $f: B^2 \to B, f(x_1, x_2) = x_1.x_2'$ **05**

OR

a) Check the validity of the following argument:
Hypothesis $S_1: p \Rightarrow (\sim q), S_2: r \Rightarrow q, S_3: r$
Conclusion: $S: \sim p$

b) Attempt any two: **10**

1) Show that $\{p \Rightarrow (q \Rightarrow r)\} \Rightarrow \{(p \Rightarrow q) \Rightarrow (p \Rightarrow r)\}$ is a tautology.
2) Using truth tables prove that $p \vee (q \wedge r) = (p \vee r)$.
3) Express Boolean function f(a, b, c)=(a . b) + (a . c) + (b . c) as a product of sums in three variables.
4) For the element x, y of a Boolean algebra, prove that $x.y' = 0 \Leftrightarrow x.y = x$